Traffic Simulation on a Two-Way Street

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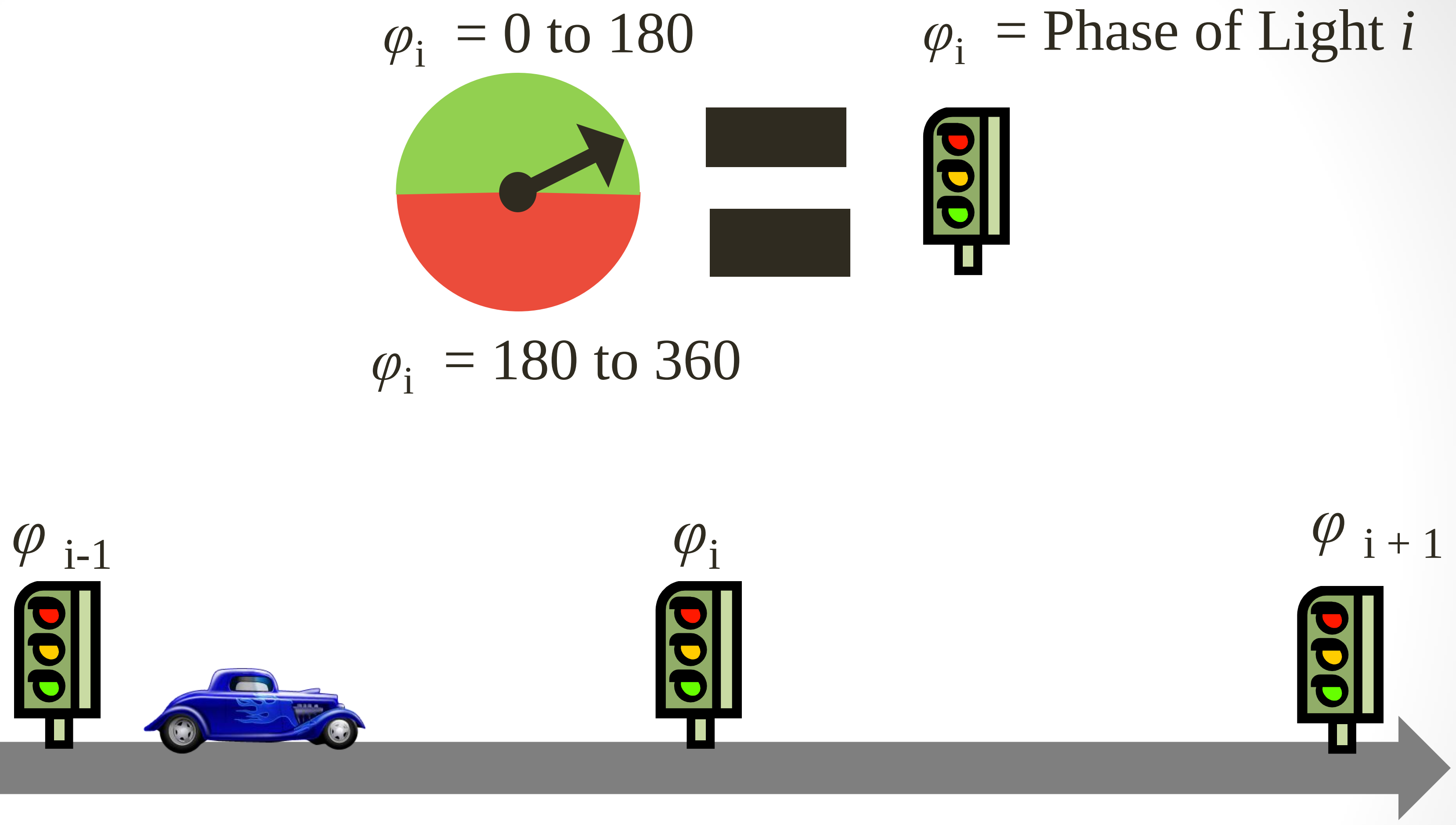
Additional Help from Mark Panaggio

And additional thanks to the Mellon Mays Fellowship


φ_i = 0 to 180
φ_i = Phase of Light *i*
φ_i = 180 to 360
φ_{i-1}
φ_i
φ_{i+1}

SOME PARAMETERS

T_C = Travel time between lights
T_L = Full period of light
T_d = Time between greens for consecutive lights
(assumed to be constant)

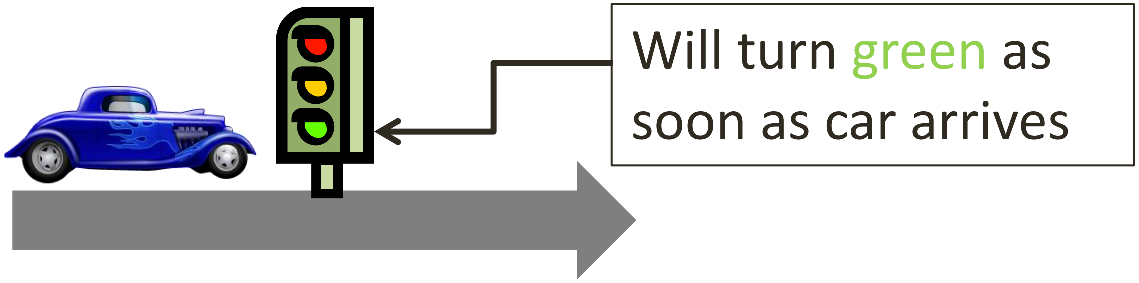
EQUIVALENTLY

$\omega = 2\pi/T_L$ = Angular velocity of lights
$\Delta\varphi = -\,\omega * T_d$ = Phase difference between consecutive lights

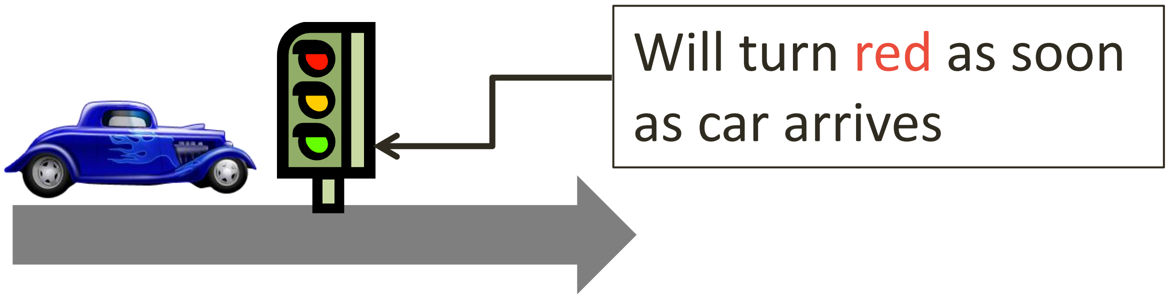
REDUCES TO

$r = T_L / T_C$
$r_d = T_d / T_c$ (so $0 \leq r_d \leq r$)

If $r_d = 1$ (i.e., $T_d = T_C$), "Green Wave"



If $r_d = r/2+1$ (i.e., $T_d = T_C + T_L/2$), "Red Wave"



Summary	Green Wave	Red Wave
Forward Direction	$r_d = 1$	$r_d = r/2 + 1$
Reverse Direction	$r_d = r - 1$	$r_d = r/2 - 1$

ANALYTIC SOLUTION

<Velocity> = Velocity * Time Spent Moving / Total Time

If we let

NLT = Number of lights the car passes between stopping red lights

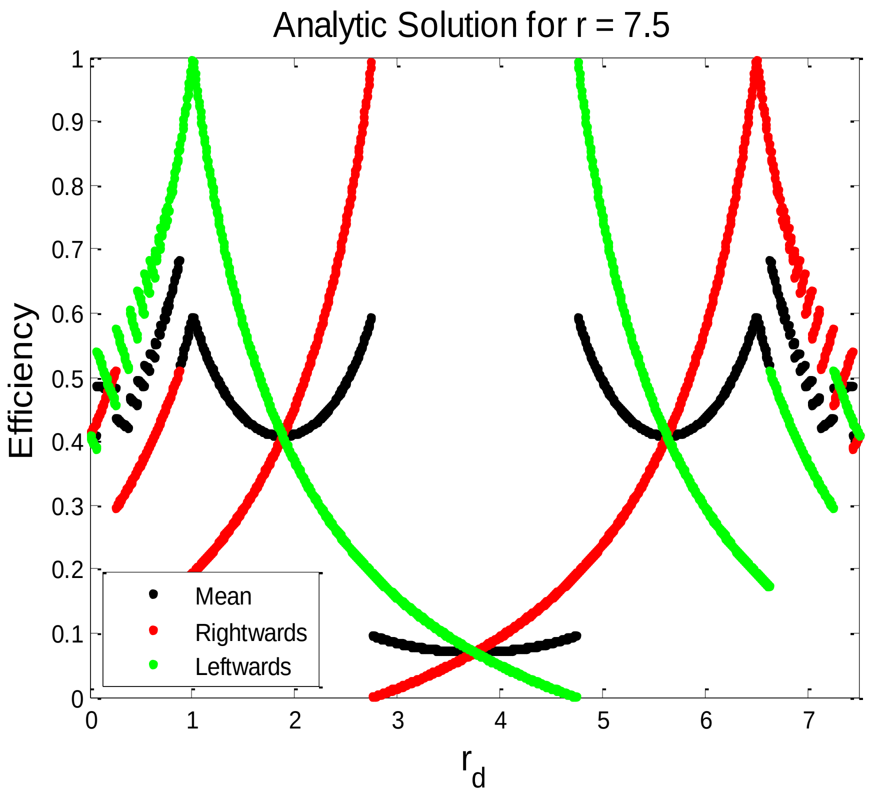
$M = r / (1 - r_d)$

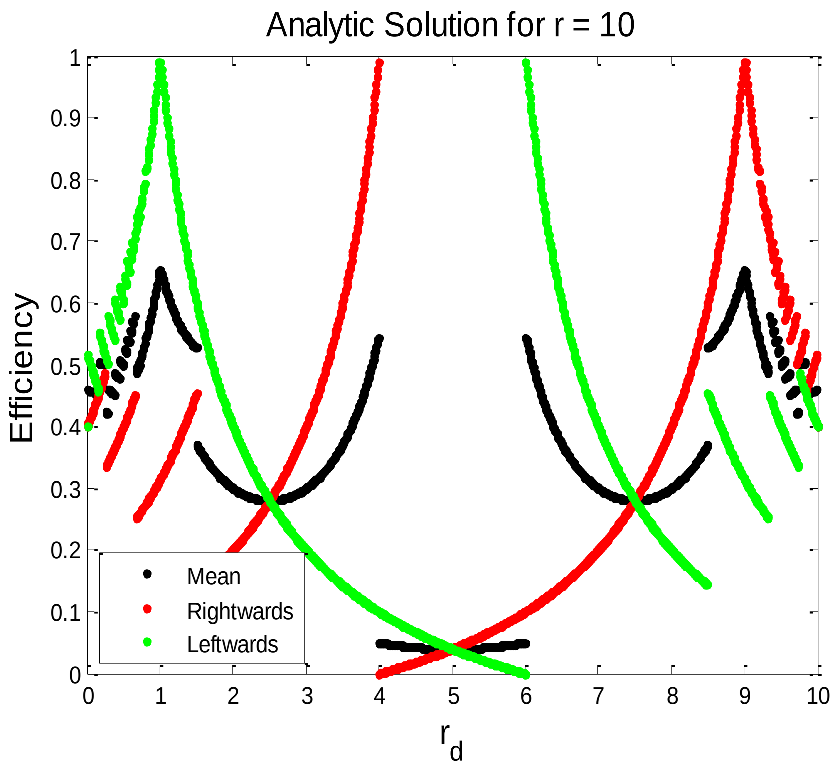
Then

$$\langle Velocity \rangle = \frac{V * NLT}{r * \mathrm{ceil}(NLT/M) + r_d * NLT}$$

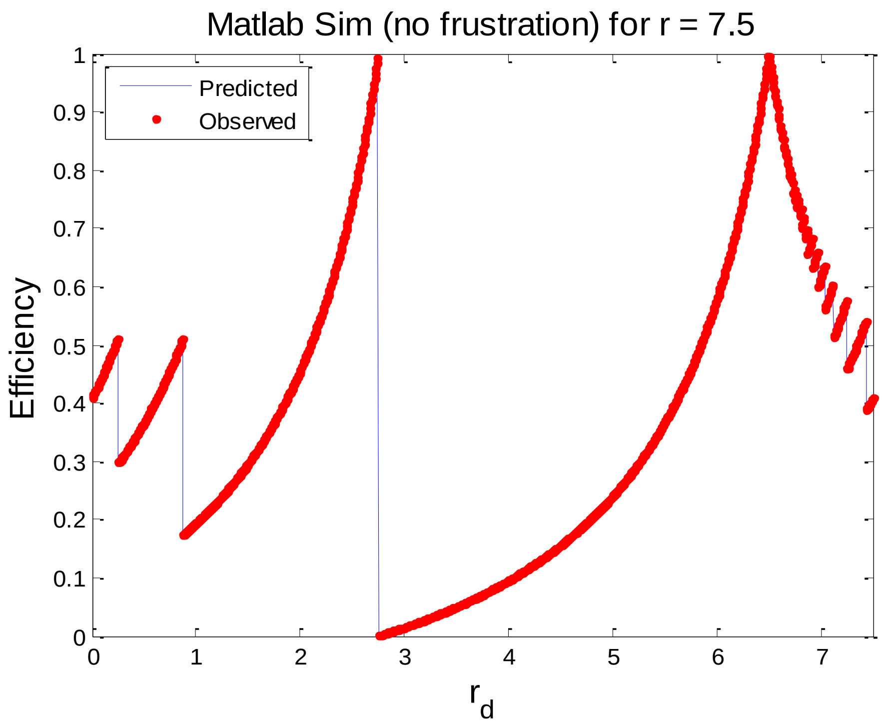
Where *NLT* is a function of *M*

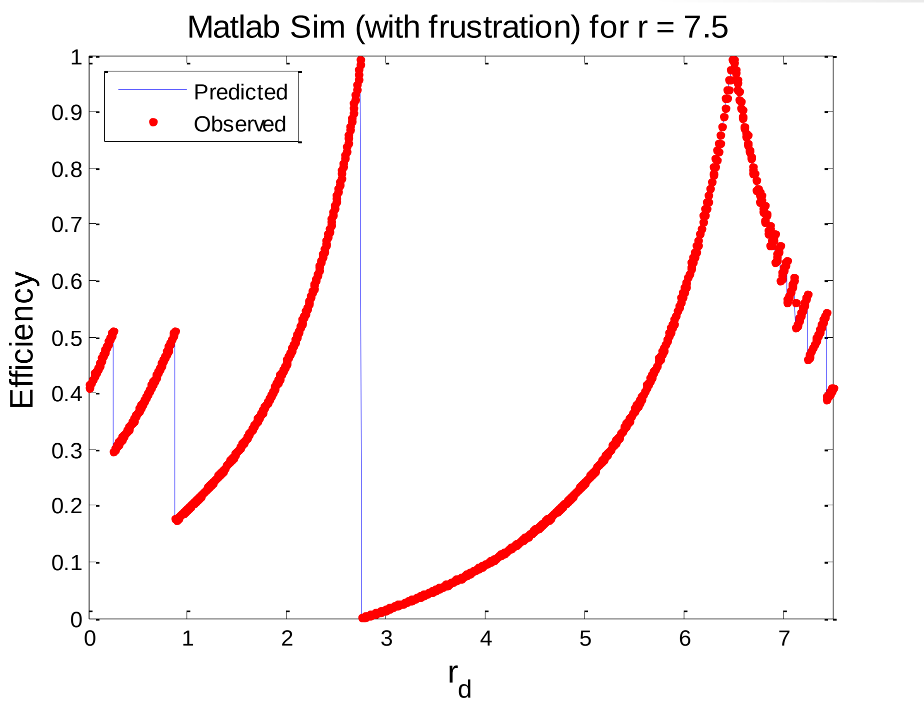
Efficiency!


Analytic Solution for r = 7.5
Efficiency
r_d
Mean
Rightwards
Leftwards
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
1
2
3
4
5
6
7

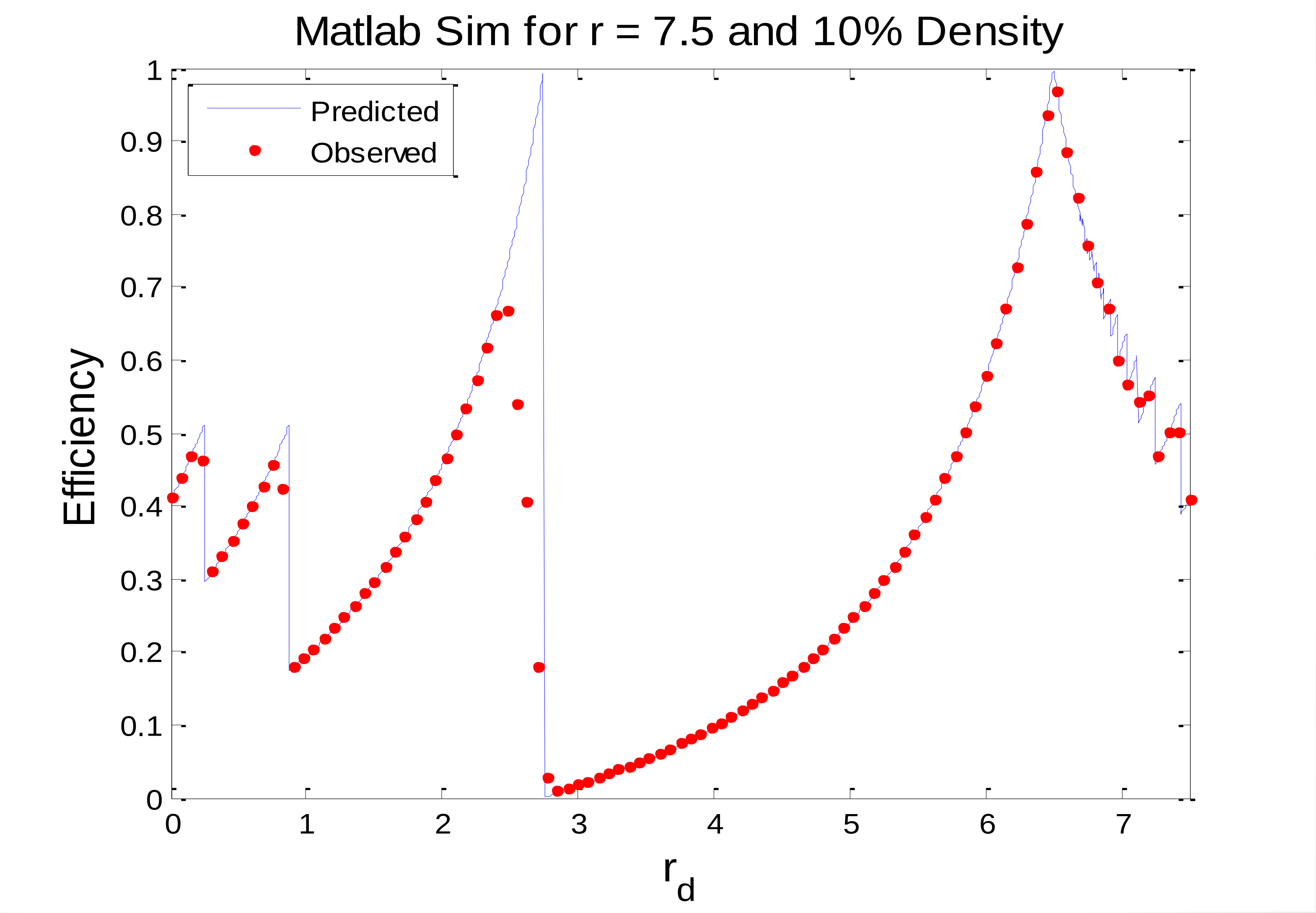

Analytic Solution for r = 10
Efficiency
r_d
Mean
Rightwards
Leftwards
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
1
2
3
4
5
6
7
8
9
10

Simulations and Edge Effects?

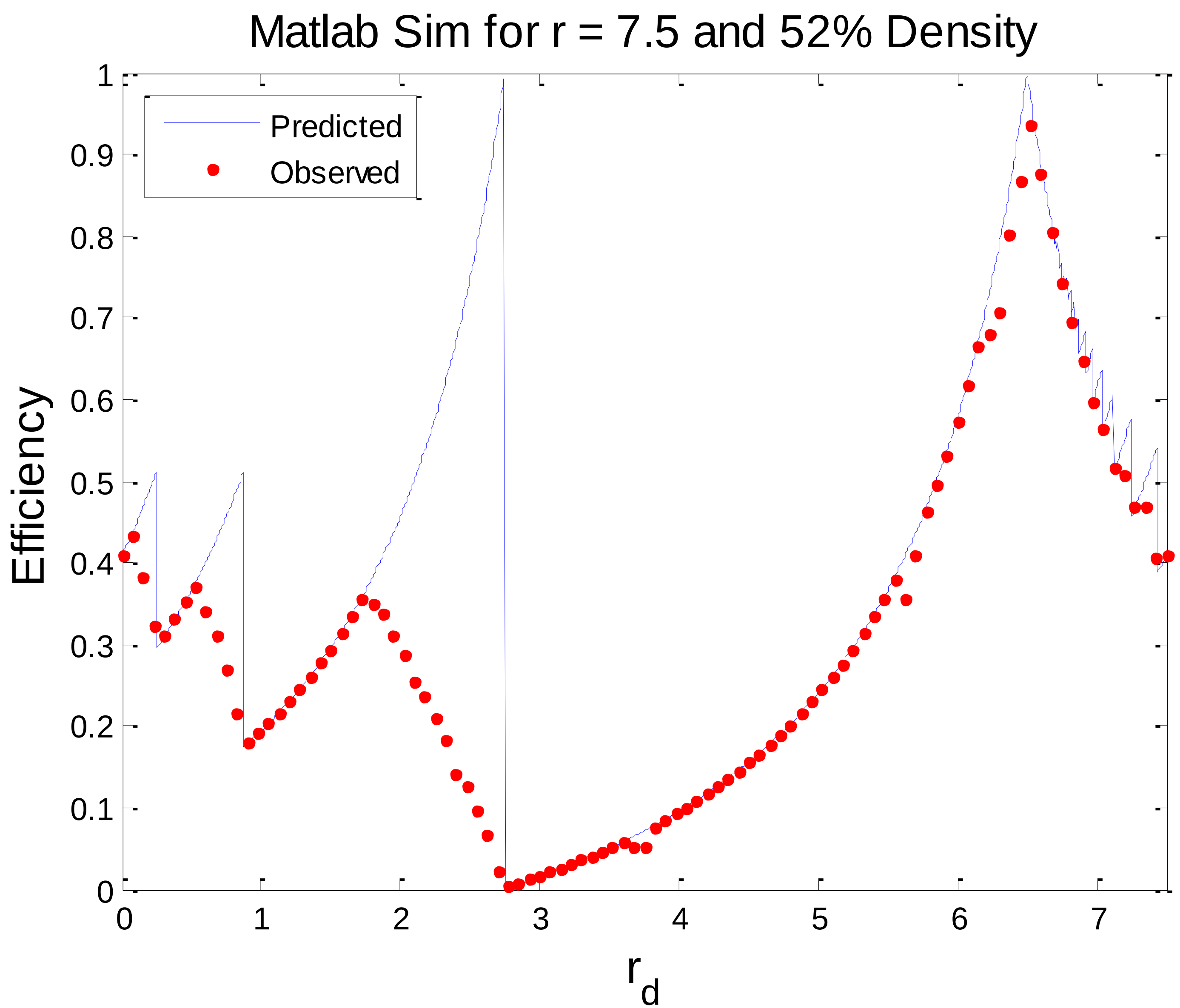




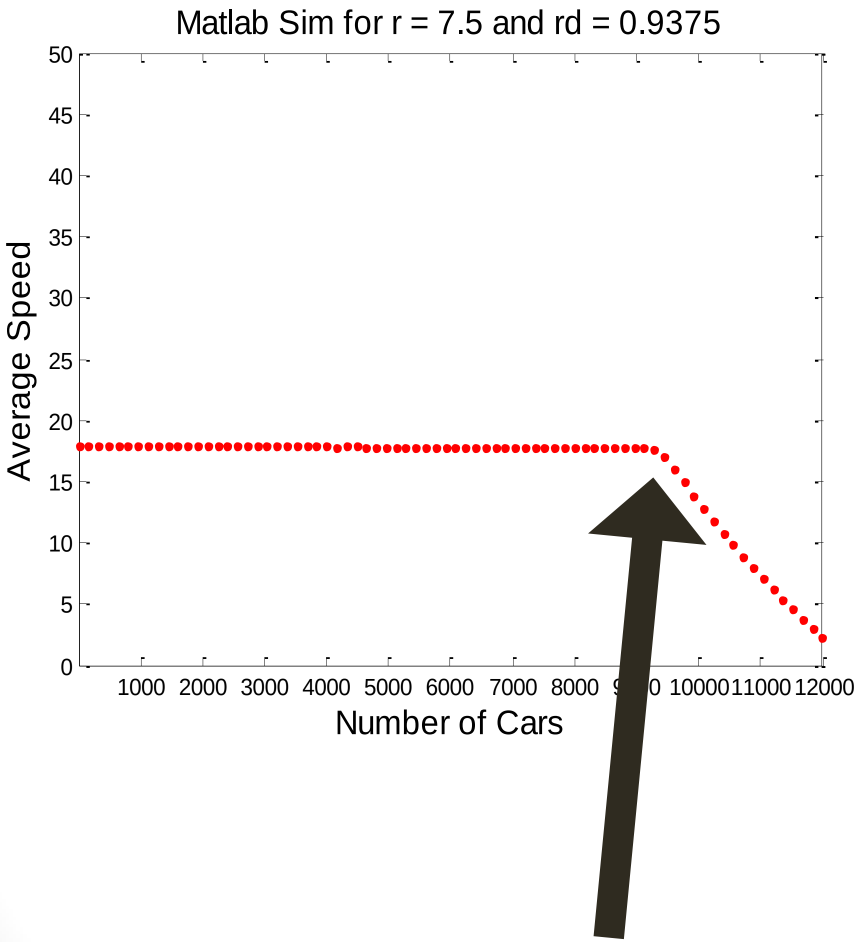
Fermionic Cars?

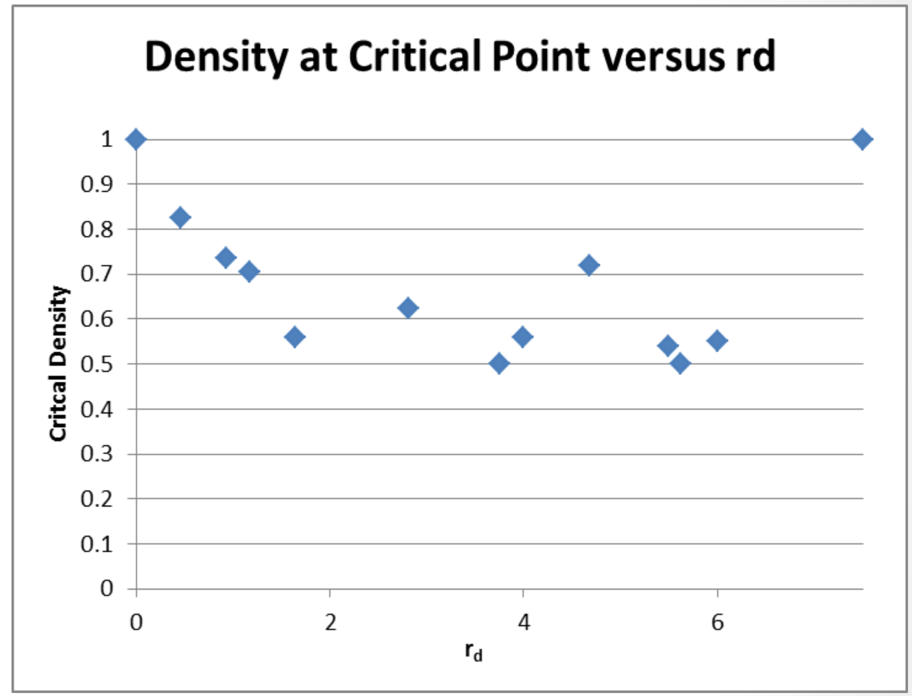


Some Values are More Sensitive.



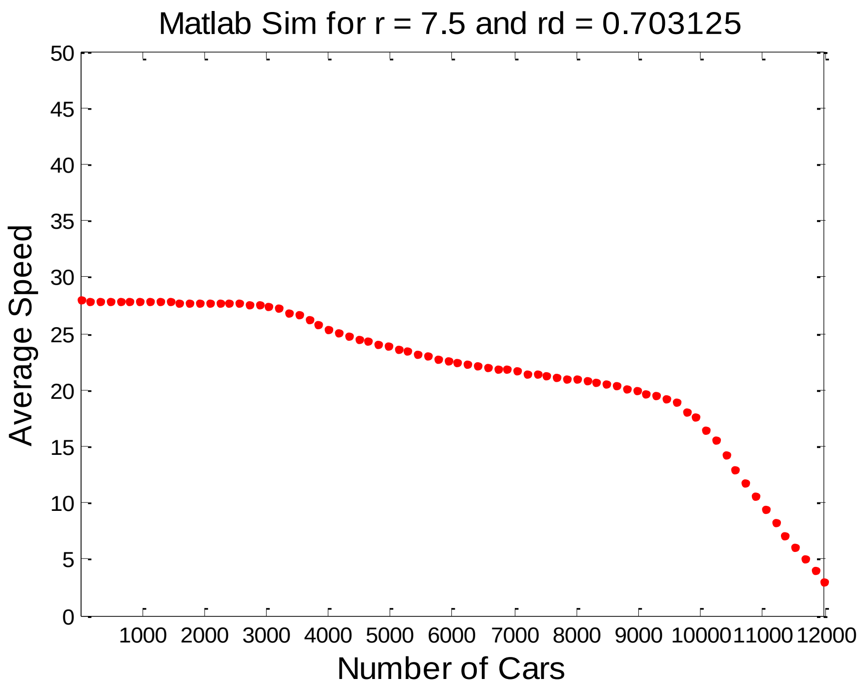
Varying Density versus Average Speed

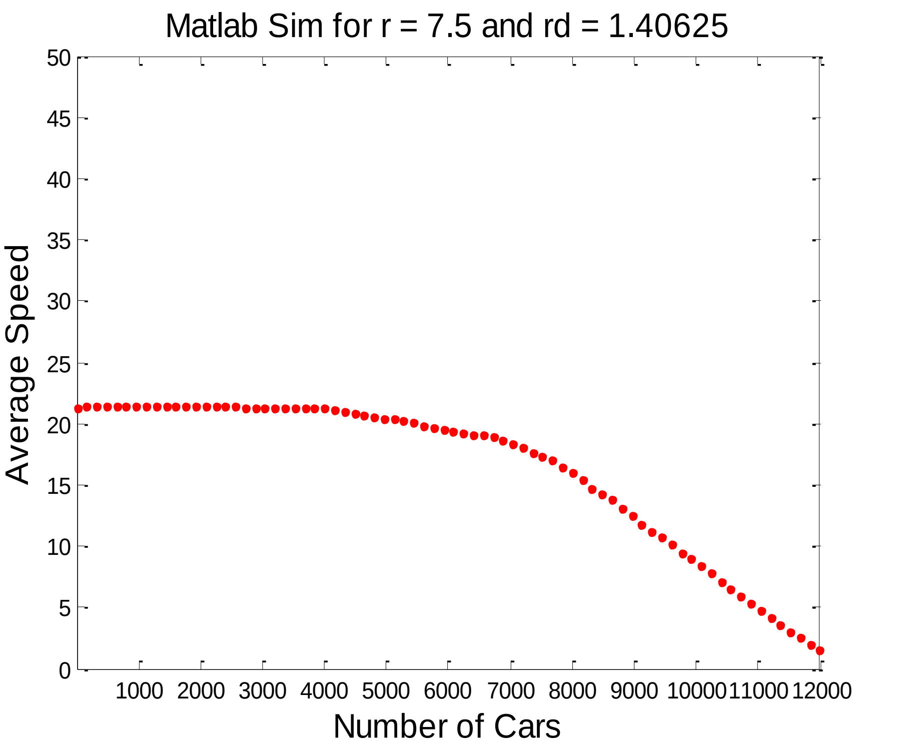




We Find Critical Densities

Critical Points Aren't Dependable





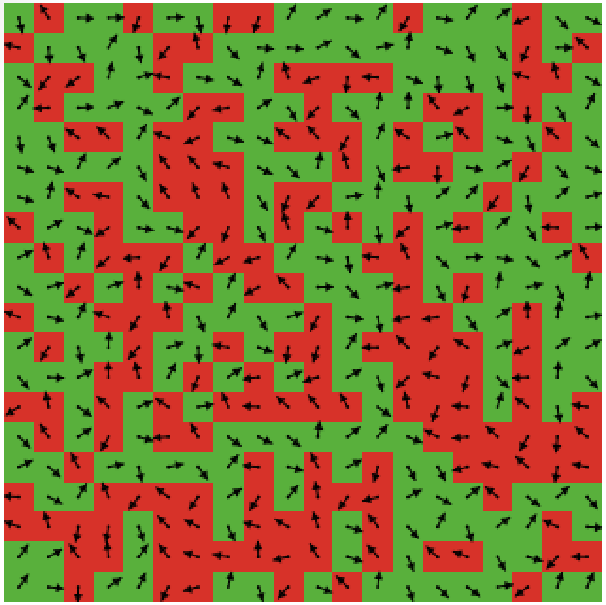
Something Completely Different

Our lights are oscillators
So why don't we couple them?

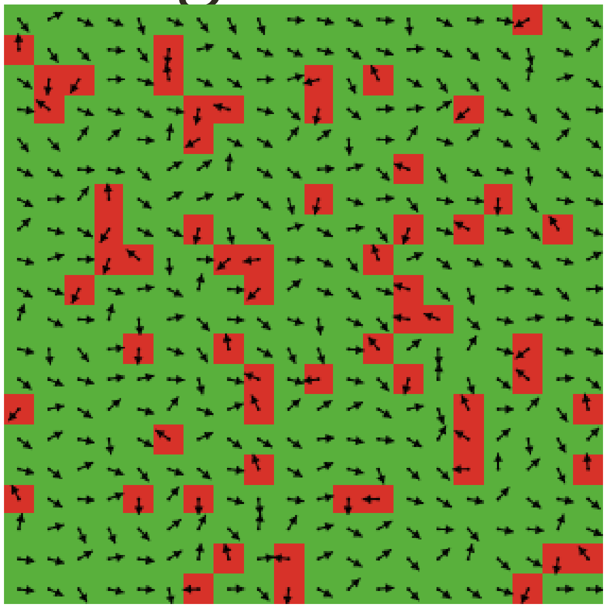
Kuramoto Coupling: $$\frac{d\phi_i}{dt} = \omega_i + \frac{K}{N}\sum_{j=1}^{N} \sin(\phi_j - \phi_i + \alpha)$$

K = Coupling strength
N = Number of lights
α = Phase delay
ϕ_i= Phase of light *i*
ω_i= Angular velocity of light *i*

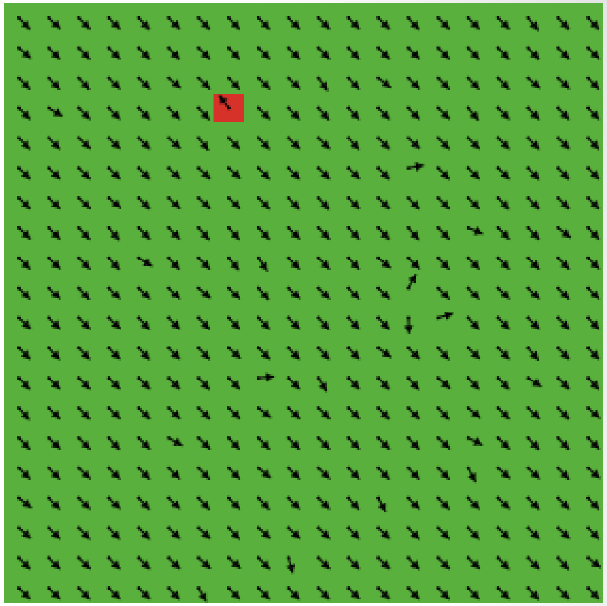
Uniform ω is boring



t = 0



t = 250



t = 500

…But what if we introduce decay factor?

$$\frac{d\phi_i}{dx} = \omega_i + \frac{K}{N}\sum_{j=1}^{N} \mathbf{G}(\mathbf{x} - \mathbf{x}') * \sin(\phi_j - \phi_i + \alpha)$$

“Chimera States” on a 1D ring

-- Coherent population moves as one

-- Incoherent population moves (almost) randomly

-- In the future, we will apply this to the ring of lights and see what happens.

References

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