Population extinction on a random fitness seascape

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Diversity of population models

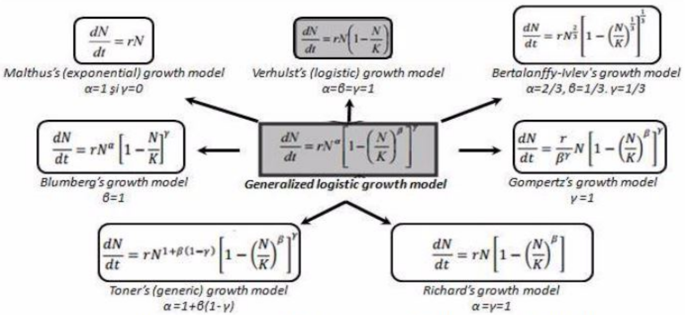


FIG. 3. The generalized logistic curve and its derivative models [11]

Figure: Cioruța (2016)

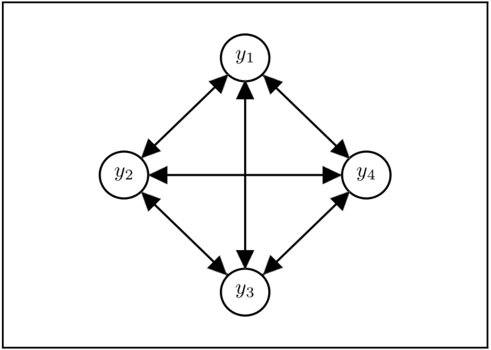
The Fisher Equation

$$\partial_t y = \mu y - a y^2 + D\nabla^2 y$$

- μ sets growth rate
- a sets saturation population.
- $D\nabla^2 y$ is diffusion in space.

The (Mean Field) Fisher Equation

$$\partial_t y = \mu y - a y^2 + D(\bar{y} - y)$$

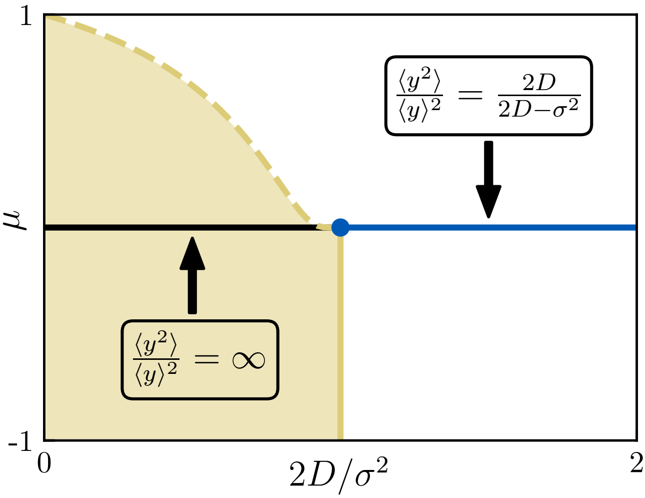


Fisher with **Seascape Noise**

$$\partial_t y = \mu y - a y^2 + D(\bar{y} - y) + \boxed{\sigma y \eta}$$

- $y\eta$ is **seascape noise**
- Represents randomness in fitness and enviroment

Universality transition



Power laws near extinction

The variance scales anomalously at high noise:

$$\langle y^2 \rangle \propto \begin{cases} \bar{y}^2, & \sigma^2 < 2D; \\ \bar{y}^{1+2D/\sigma^2}, & \sigma^2 > 2D. \end{cases}$$

Fisher Averaged leads to Richard's Equation

The Richard's Equation

The Richard's Equation is given by

$$\partial_t y = \mu y - a y^\alpha,$$

where typically $1 < \alpha \leq 2$.

Our Results

Near extinction, the mean of the Stochastic Fisher Eq. behaves as

$$\begin{aligned} \partial_t \bar{y} &= \mu\langle y\rangle - a\langle y^2\rangle + D\langle \bar{y} - y\rangle + \langle \sigma y \eta\rangle \\ &= \mu\bar{y} - a\langle y^2\rangle \\ &= \mu\bar{y} - a\bar{y}^{1+\beta}, \end{aligned}$$

where $\beta = \min\left(1, 2D/\sigma^2\right)$.

Agreement

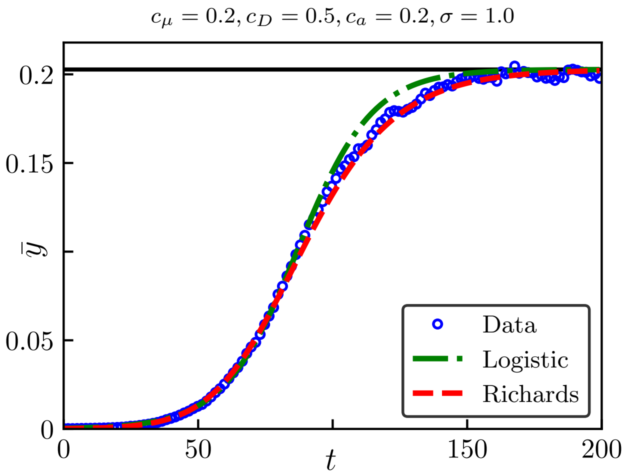


Figure: $c_\mu = 2\mu/\sigma^2$, $c_D = 2D/\sigma^2$, $c_a = 2a/\sigma^2$.