

A Richards-Like Stochastic Population Growth Model

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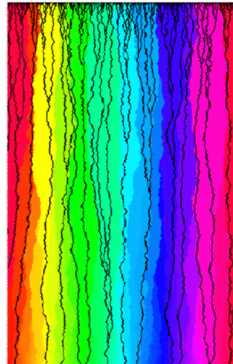
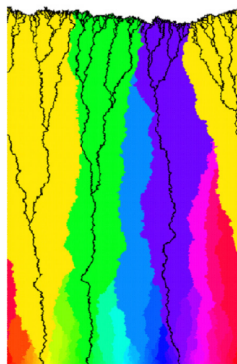
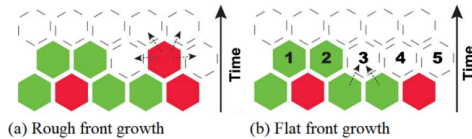
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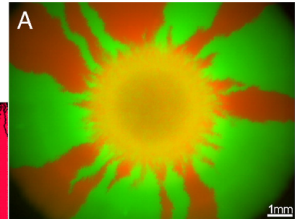
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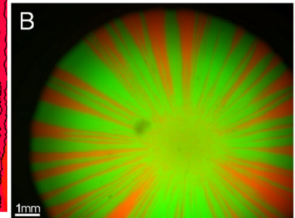
Population Growth With Randomness¹



E. coli



S. cerevisiae



¹Images: S. Chu (2019), K. S. Korolev (2010), Hallatschek (2007).

General Population Growth Model

$$\dot{y}(\mathbf{x}, t) = \mu y - ay^\alpha + D\nabla^2 y + \sigma y \eta(\mathbf{x}, t)$$

- $\dot{y} = \mu y - ay^\alpha$ is the Richards equation, as used in generalized logistic growth models.
- $D\nabla^2 y$ adds spatial diffusion.
- $\sigma y \eta(\mathbf{x}, t)$ introduces randomness proportional to population.

$\mu \neq 0$ is Simple

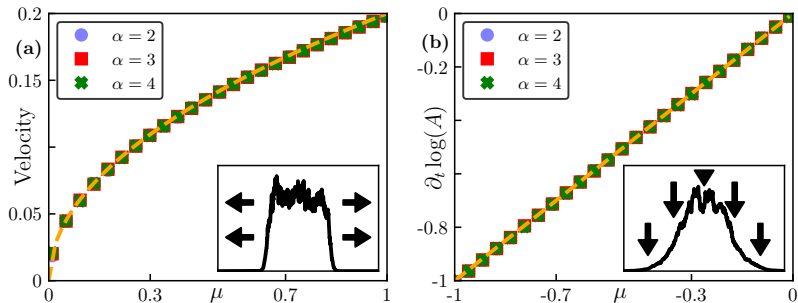
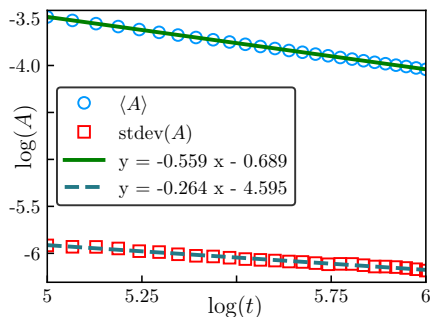


Figure: (a) $\mu > 0$, (b) $\mu < 0$.

Behavior at $\mu = 0$ Still Complicated



- What is the final stationary distribution for $\log(y)$ across space? Across realizations?
- How reasonable is it to perturb off of the noise-free ($\sigma = 0$) case?
- Does the roughness produced under Richards-like nonlinearity fall under KPZ universality?