Spatial Competition with Interacting Agents

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Abstract

We generalize Hotelling's model of spatial competition with more than two firms in a two-dimensional space. Firms choose both price and location to maximize profits. The principle of minimum differentiation does not hold in general. Local duopolies emerge from the interaction between firms. Firms do not spread uniformly across the two-dimensional space, nor do they all charge the same price. Firms in more competitive locations charge lower prices. More product attributes produce more price competition.

JEL Classification D21, D43, D83.

Key Words Hotelling, Spatial Competition, Product Differentiation, Prices, Principle of Minimum Differentiation.

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1 Introduction

In a pioneering paper Hotelling (1929) presented a model of two firms competing to sell a homogeneous product to customers uniformly distributed along a line. Though the Hotelling model has been extended in many ways[1], no one has produced a general model of spatial competition between more than two firms in a two-dimensional space, with both price and location as choice variables. The problem is important because competition in the real world seldom occurs between two firms on a one-dimensional space. Most products have multiple attributes and multiple firms supplying them.

In this paper, we develop an agent-based simulation model to study spatial competition in a two-dimensional space with multiple firms. We develop an agent-based model because it is difficult to generalize the Hotelling model using analytical methods. Interaction between firms in a two-dimensional space makes it difficult to compute the areas served by different firms. The problem has so far proved analytically intractable. In the words of mathematical economists Eaton & Lipsey (1975, 41):

> To determine if the configuration is an equilibrium one with respect to a small movement of the firm in any direction is an almost impossible task using analytical methods, and in any case much more is required to establish global equilibrium.

Eaton & Lipsey (1975) built one of the first agent-based simulation models to pursue the problem of spatial competition. They located a handful of firms on a space and numerically calculated the market area for each firm for a number of alternative locations. The technical limitations of the age meant that they could study only a few moves by each firm. We develop Eaton and Lipsey's approach to produce a general model of spatial competition. Firms are placed on random locations on a two-dimensional grid. Customers are spread evenly across the grid. Every time step, firms make price and location choices. Firms change price or location if it is profitable to do so. The principle of minimum differentiation does not hold in general. Interactions between firms produce local duopolies, with intense local competition within coupled-firms and more stable division of customers between different couples. This is akin to how large home improvements stores like Home Depot and Lowes compete on

[1] Economists have applied Hotelling's model to study a wide variety of phenomenon including political competition (Roemer 2009), agglomeration in cities (Fujita & Thisse 2013), product differentiation (Meagher & Zauner 2004), price-increasing effects of competition (Chen & Riordan 2008), the dynamics of resource extraction (Lin & Wagner 2007), and the relation between prices and market integration (Asche, Bremnes & Wessells 1999). Theoretical work has extended Hotelling's model to include uncertainty (Meagher 2012), transportation costs (Anderson 1988), different distributions of customers (Gupta, Pal & Sarkar 1997), endogenous number of firms (Götz 2005), and elastic demand for product (Puu 2002).

prices, and smaller hardware stores appeal to niche markets by locating in isolated places or delivering specialized products.

Our paper is related to decades of work on generalizing Hotelling's model. Devletoglou (1965), Jonas (1968), Drezner (1982), Tabuchi (1994), and Veendorp & Majeed (1995) study the problem of two firms on a two-dimensional space. All but Tabuchi (1994) assume both firms charge the same price. Tabuchi uses an astute transformation to study price and location choices. However the transformation is not valid for systems with more than two firms. Wendell & McKelvey (1981), Ben-Akiva, De Palma & Thisse (1989), and Okabe & Aoyagi (1991) study a system with more than two firms in a two-dimensional space. Wendell & McKelvey (1981) explore a variety of symmetric equilibria using a 'guess and check' method under the assumption that all firms charge the same price. Ben-Akiva, De Palma & Thisse (1989) allow firms to charge different prices but fix location along one dimension. Okabe & Aoyagi (1991) allow firms to choose location along both dimensions but assumes all firms charge the same price. Irmen & Thisse (1998) and Hehenkamp & Wambach (2010) study competition between two firms in higher dimensional spaces. They characterize a subset of equilibria for which analytical solutions exist. Caplin & Nalebuff (1991) prove the existence of equilibrium for any number of firms in high dimensional spaces. But they assume location is not a choice variable.

Most authors assume either price or location is fixed. Despite the simplification, they often study a subset of equilibria using 'guess and check' methods. Though some equilibria can be characterized by the 'guess and check' approach, it is difficult to say whether these represent a non-trivial portion of the universe of equilibria. Furthermore, there is little reason to believe that the equlibria which are the least difficult to characterize are economically the most relevant.

Overall, it would be fair to say that the problem of spatial competition with both price and location as choice variables remains open. If a good many mathematical economists are to be believed, then the problem of computing analytical solutions of a general model of spatial competition is "very tedious" (Irmen & Thisse 1998, 84), perhaps "impossible" (Jonas 1968, 17). Future developments in mathematical methods may allow economists to develop analytical solutions of general spatial competition. But we need not sit idle in the meanwhile. The mathematical intractability of the problem begs for a whole new approach. Agent-based modeling is one such approach. Recent developments in computing technology allow us to build and study synthetic economies populated with artificial agents *in silico* (Axtell 2000). Some consider agent-based modeling as a 'new mathematics' for social science (Borrill & Tesfatsion 2011). Others have called it a 'third way of doing science' because it contains elements of deduction and induction (Axelrod 1997). We present an agent-based implementation of the Hotelling model with many firms in a two-dimensional space. In addition to characterizing equilibria, our model sheds light on out-of-equilibrium dynamics. The paper is organized as follows. Section 2 develops the model. Section 3 presents the results. Section 4 offers concluding remarks.

2 The Model

There are two types of agents: firms and customers. Firms are randomly placed on points in a two-dimensional grid cell coordinate system. Each square patch in the grid is a customer. Three events occur every time step. One, each customer buys one unit of the good from its lowest cost firm. Two, firms evaluate moving to a different location in the two-dimensional space and changing their price. And three, firms simultaneously exercise their location and price choices. We run the economy forward in time and analyze the resulting data. The model is written in the NetLogo agent-based modeling language (Wilensky 1999). The model is freely available from Netlogo's model library http://ccl.northwestern.edu/netlogo/models/Hotelling'sLaw (Ottino, Stonedahl & Wilensky 2009).

2.1 Firms

The cost of production is zero, profit equals revenue. Firms begin with randomly assigned locations and price of ten. Every time step, firms compute the profit from a one unit move in each of the four cardinal directions. They move to the point with the greatest profit. Firms do not move if the profit at the present location is greater than the profits at any of the four alternative locations. Every time step, firms compute the profit from a one unit increase and a one unit decrease in price. They charge the price with the greatest profit. Firms do not change price if the present price offers a profit greater than the alternatives.

Firms compute profits from potential changes in location and price by collecting demand information from customers. Customers truthfully report whether they would buy from a firm given its location and price. Firms incur no cost of moving locations, changing price, or collecting information. While making location and price decisions, each firm assumes that all other firms will remain at the current locations and charge existing prices. In other words, we assume 'zero conjectural variation'.

2.2 Customers

Every time step, customers compute the cost of buying from all firms on the grid. The cost of a good is the sum of the price charged and the distance to the firm. The distance equals the Euclidean norm. Every time step, each customer buys one unit of the good from its lowest cost firm. When indifferent between multiple firms, customers randomly select one firm from the indifference set.

2.3 Model visualization

Visualization is an important part of agent-based models. Visualization serves as an aid for understanding and analyzing model dynamics. In our model, firms are represented by circles, each filled in with a distinct color. The grid cells, each representing a customer, are colored using a lighter shade of the same hue as the store which they currently patronize. This produces the appearance of a continuous filed partitioned into different colors of shopping districts. For static visualization of out-of-equilibrium dynamics (such as figures 6 and 7), the history of movements of each store is displayed using a trail of corresponding color. While static visualization does not fully capture the time dynamics of the model, it does show the general nature of movements. The following section summarizes the data from five hundred computational experiments run on a 41x41 grid cell coordinate system with 160 customers.

3 Results

Result 1. *The principle of minimum differentiation does not hold in general.*

Hotelling (1929) found two firms competing on a line choose to locate next to each other. This came to be known as the principle of minimum differentiation. Hotelling himself, and many since, claimed the principle explains a variety of phenomena including why political parties are all alike (Boulding 1955). In our model, the principle of minimum differentiation holds when there are two firms. But the principle does not hold when there are three or more firms. Previous economists have shown the principle of minimum differentiation is sensitive to the assumptions of Hotelling's model. In particular, economists have proved that the principle does not hold if demand is price elastic (Lerner & Singer 1937, Robinson 1941, Smithies 1941, Devletoglou 1965, Hartwick & Hartwick 1971) or if transportation costs are non-linear (d'Aspremont, Gabszewicz & Thisse 1979). Unlike previous work, we show that the principle of minimum differentiation does not hold even with Hotelling's original assumptions. Like Hotelling's model, our model assumes demand is price inelastic and transportation costs are linear.

The principle of minimum differentiation does not hold for the following reason. Imagine a system with two firms: i and j. As i moves towards j, i will gain some of j's customers while retaining all of its own customers. Introduce another firm k into the system. With three firms, as i moves towards j, i will gain some of j's customers but may lose some of i's own customers to k.

At the end of every simulation, the distance between each firm and its nearest competitor is recorded. Figures 1 and 3 plot the minimum distances, while figures 2 and 4 plot the maximum distances at the end of every simulation. Figures 1 and 2 pertain to the case when firms choose locations but prices are fixed. Figures 3 and 4 pertain to the case when firms choose both locations and prices. In figures 1, 2, 3, and 4 with two firms the minimum and maximum distances are either 0 or 1. The two firms are either on the same location or adjacent to each other. Firms in our model seek local improvements assuming their competitor will maintain price and location. The two firms pivot around each other producing a distance of 1 in some cases and 0 in others. With more than two firms, firms arrange themselves in many different constellations. Figure 2 and 4 show firms do not form one large cluster.

Result 2. *Local duopolies emerge from the interaction between firms.*

An emergent property of system is that firms form pairs, but never triples or groups of other sizes. Our result is similar to that of Eaton & Lipsey (1975). Firms have a tendency to move towards another firm near them. However, pairs of firms do not together move towards other pairs. Rather they engage in intense competition with each other. Lerner & Singer (1937) and Chamberlin (1953) showed that in equilibrium some firms form pairs in the one-dimensional case. They argued firms at the edges must necessarily forms pairs because otherwise they have a tendency to approach the center. We do not find this to be true in the two-dimensional case. Singleton firms may exist in the periphery. Unlike in one-dimension, in two-dimensions a firm may loose some of its customers to other firms if it approaches the center from the periphery.

Figures 5 shows the proportion of firms that form pairs. Circles denote the case when firms choose locations but prices are fixed. Crosses denote the case when firms choose both locations and prices. In most cases, a significant proportion of the firms form pairs. Figures 6 and 7 show the characteristic spatial locations along with the path taken by the firms. Table 1 lists the prices of firms in figure 7.

Result 3. *Firms charge lower prices in more competitive regions.*

Firms with other firms near them charge low prices, whereas firms far away from competitors charge high prices. Some firms compete over large central areas by charging low prices, other firms move to the periphery to occupy smaller regions charging high prices.

Figures 8, 9, 10, and 11 plot the relation between prices and distance to the nearest competitor. Figures 8 and 10 plot the relation between minimum distance and minimum price. Figures 9 and 11 plot the relation between maximum distance and maximum price. Figures 8 and 9 present the case when firms choose prices but locations are fixed. Figures 10 and 11 present the case when firms choose both prices and locations. Figures 8, 9, 10, and 11 show that there is a positive relation between distance to the nearest competitor and price. The greater the distance to a competitor, lower the level of competition, and greater the price.

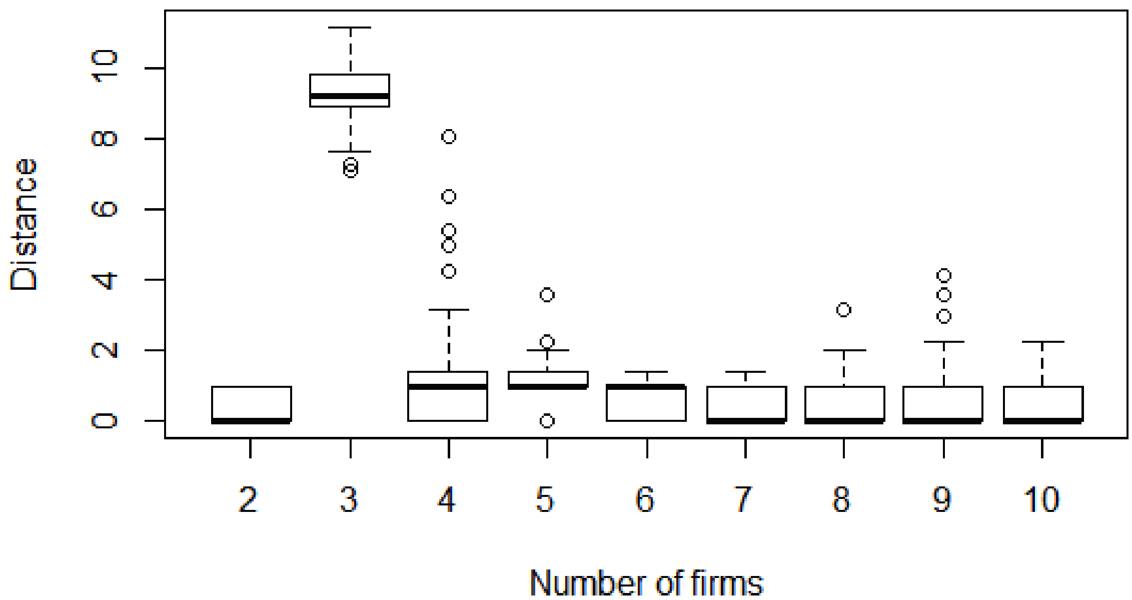

Figure 1: Minimum distance to nearest when firms choose locations but prices are fixed.

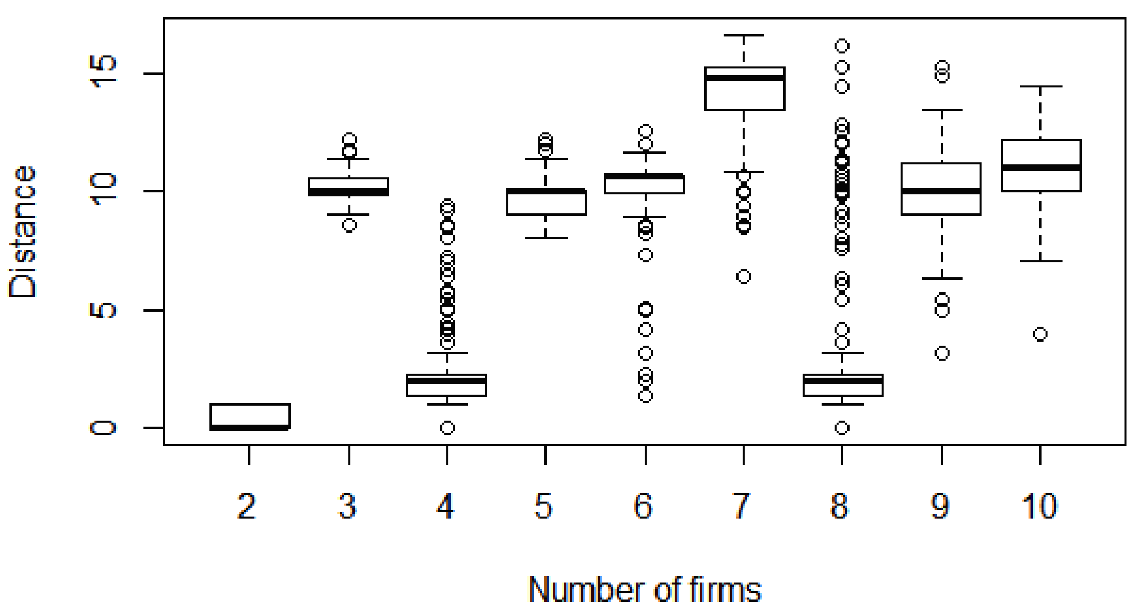

Figure 2: Maximum distance to nearest firm when firms choose locations but prices are fixed.

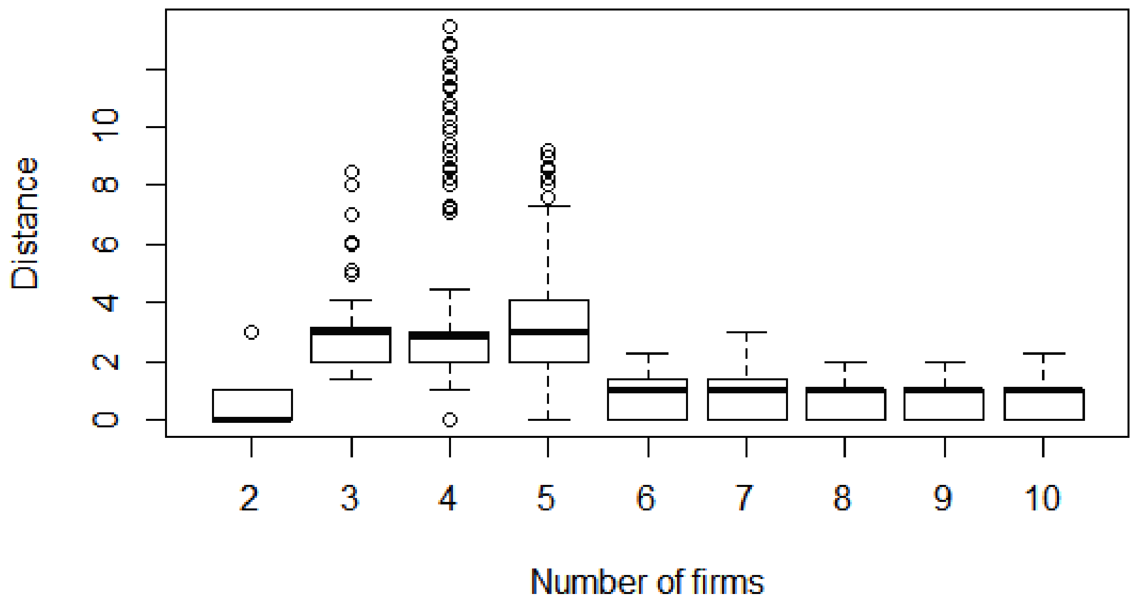

Figure 3: Minimum distance to nearest firm when firms choose both prices and locations

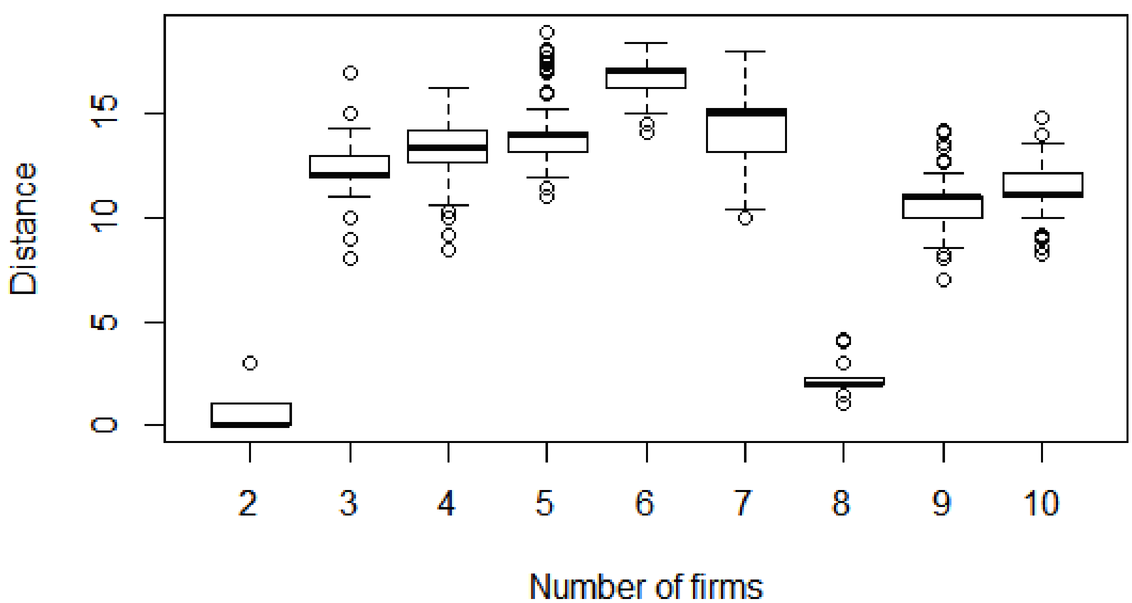

Figure 4: Maximum distance to nearest firm when firms choose both prices and locations

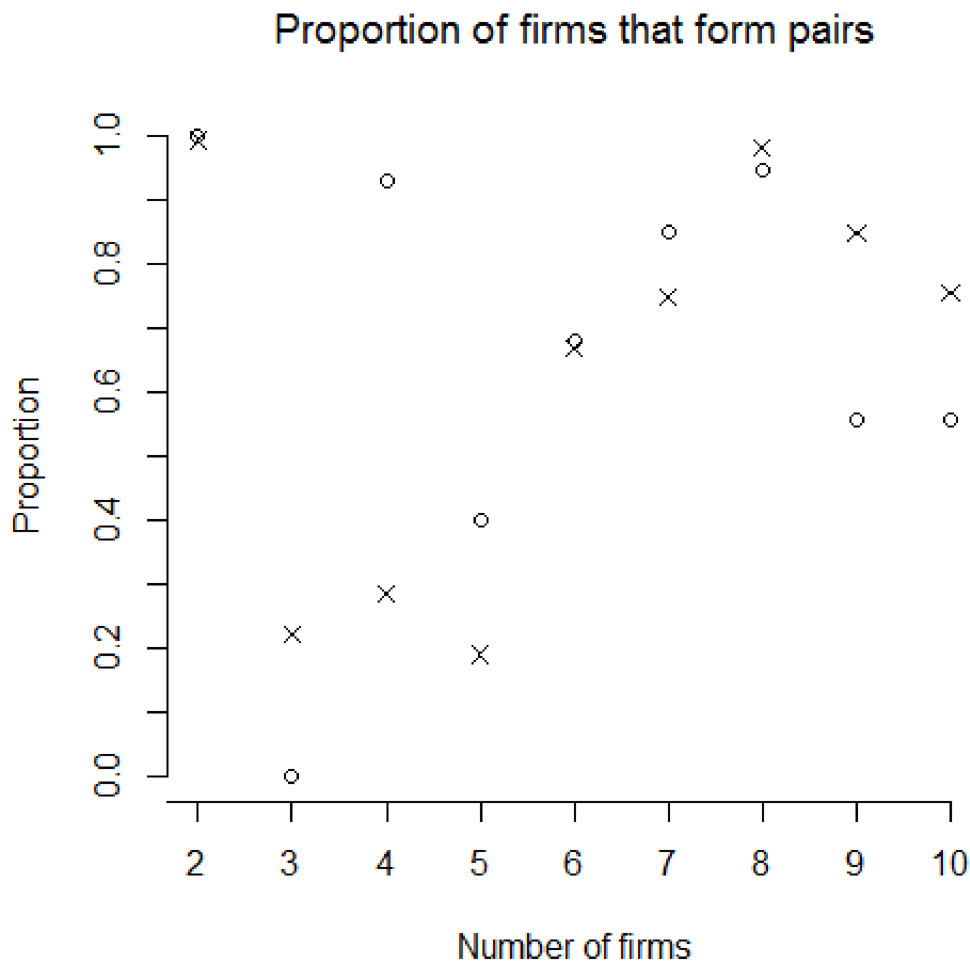

Figure 5: Proportion of firms that form pairs. Circles mark the case when firms choose locations but prices are fixed. Crosses mark the case when firms choose both locations and prices.

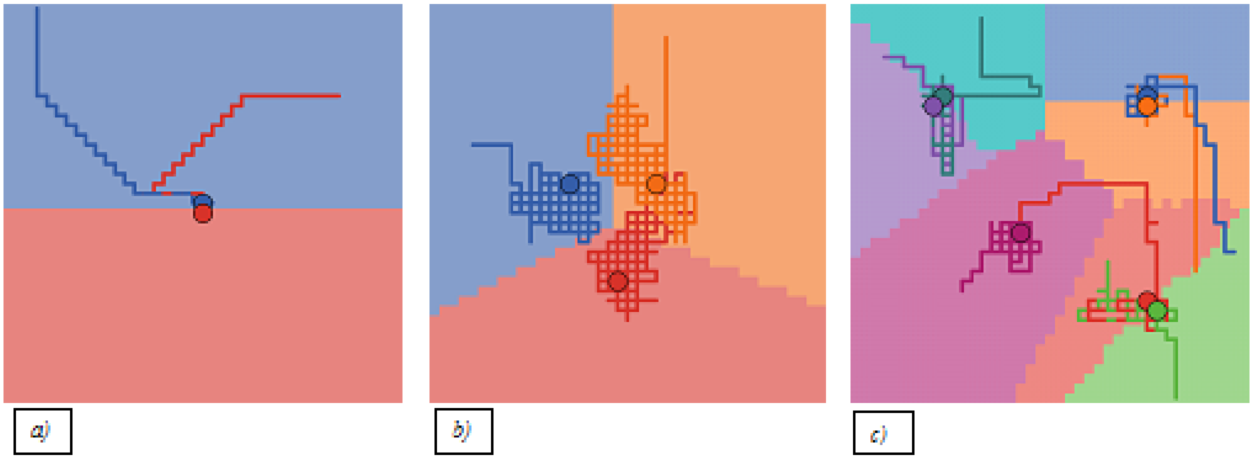

Figure 6: Characteristic locations with two, three and seven firms when firms choose locations but prices are fixed.

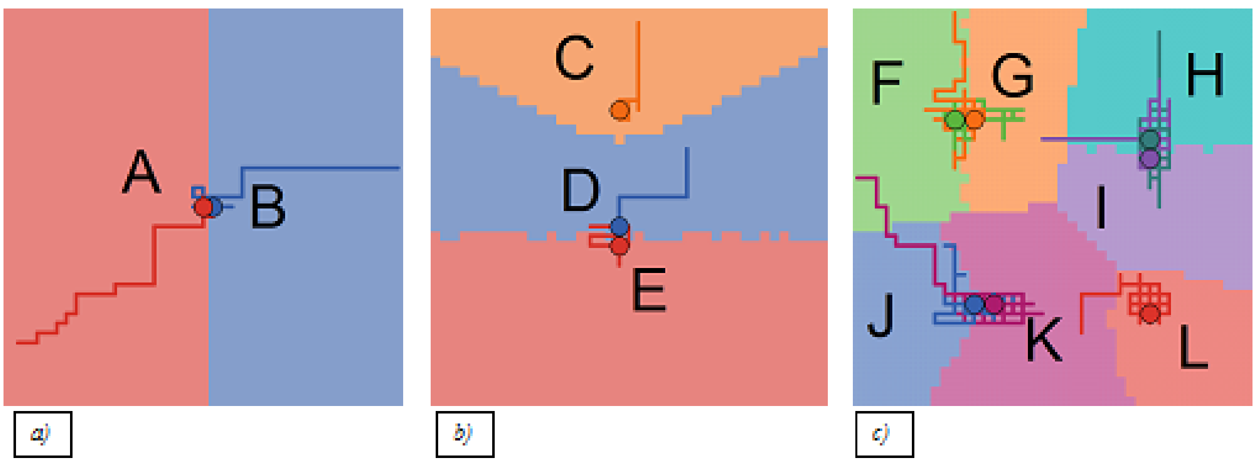

Figure 7: Characteristic locations with two, three and seven firms when firms choose both prices and locations.

Table 1: Selling prices for the stores depicted in Figure 6

Region	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**
Price Charged	1	1	8	2	2	2	2	3	3	2	3	10

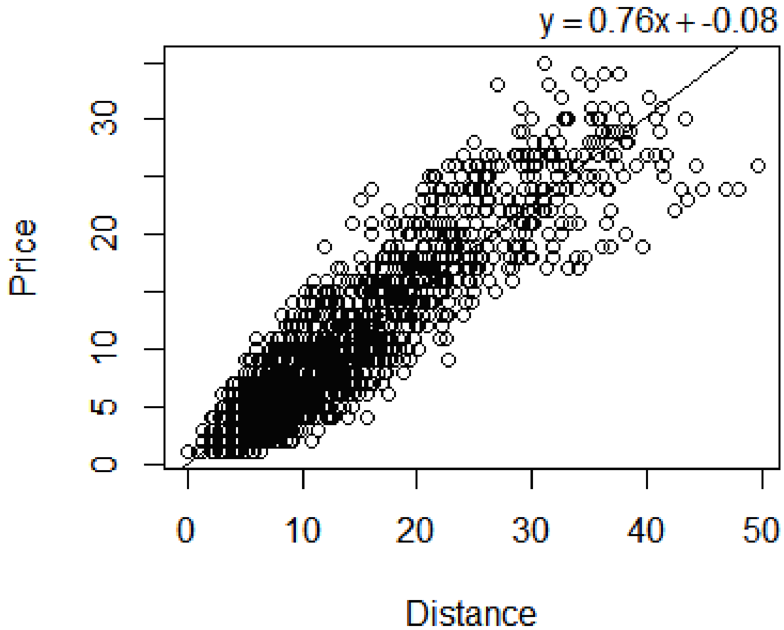

Figure 8: Minimum distance to nearest firm versus price when firms choose prices but locations are fixed.

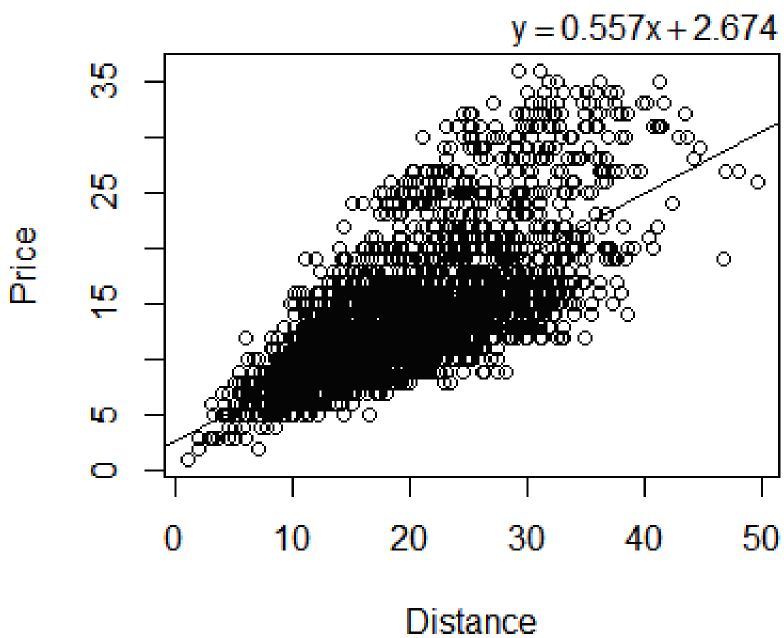

Figure 9: Maximum distance to nearest firm versus price when firms choose prices but locations are fixed.

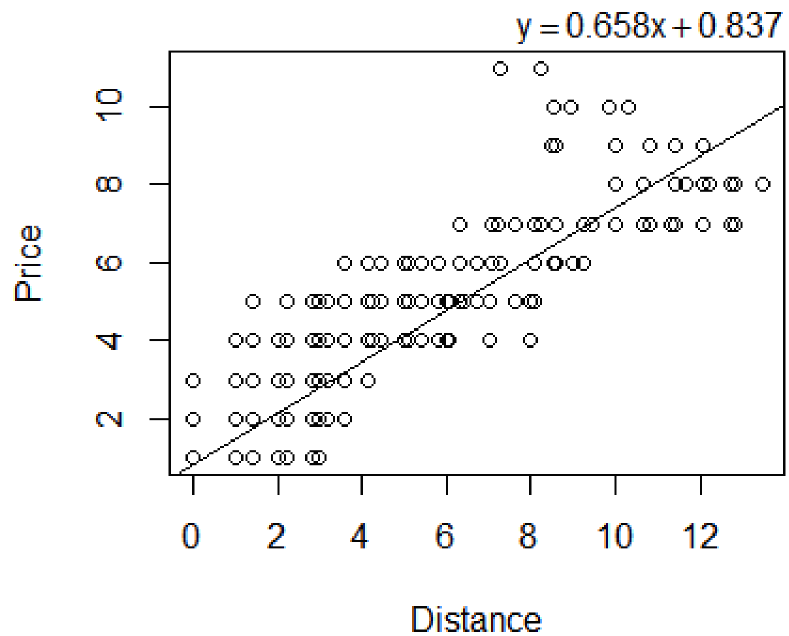

Figure 10: Minimum distance to nearest firm versus price when firms choose both locations and prices.

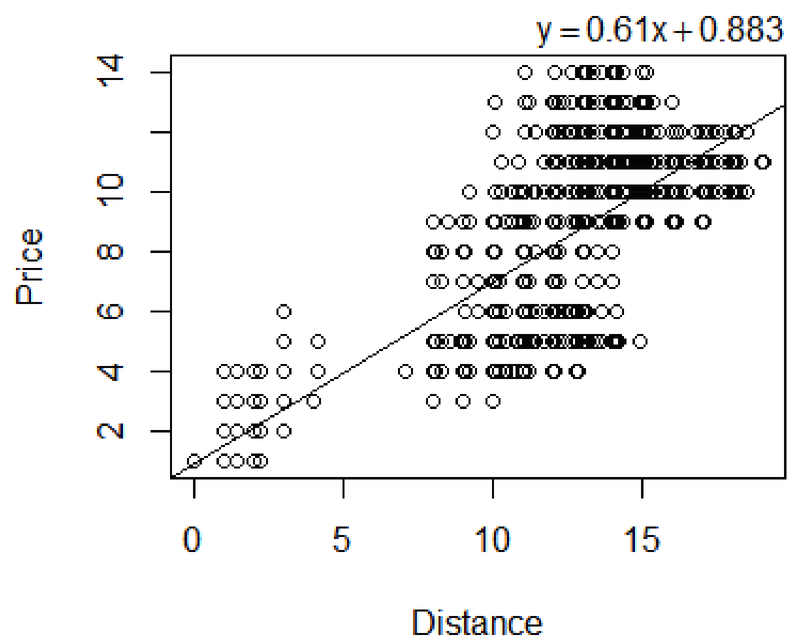

Figure 11: Maximum distance to nearest firm versus price when firms choose both locations and prices.

4 Concluding Remarks

Economists have long known that firms compete not just on prices but also on the nature of the product. The product itself is an economic variable (Chamberlin 1953). In a pioneering article, Hotelling (1929) developed a formal framework to study price and product competition. Though over 85 years have passed since Hotelling's contribution, no one has extended the model to two dimensions with multiple firms making price and location choices. The problem has proved analytically intractable. We solve the problem using agent-based simulation. Our paper shows that agent-based simulation is a general purpose technique, which can be used to solve some hard problems of economic theory.

Our model is a highly stylized depiction of the complex realities of price and product competition. The model can be extended to include demand elasticity, asymmetric information, various interaction topologies, and different distribution of customers. To pursue all, or even a few, of these facets simultaneous would be nearly impossible, possibly unfruitful. We have pursued a few limited aspects in the paper with the hope of shedding some light on the general nature of economic competition.

References

Anderson, Simon P. 1988. "Equilibrium existence in the linear model of spatial competition." *Economica*, 55(220): 479–491.

Asche, Frank, Helge Bremnes, and Cathy R Wessells. 1999. "Product aggregation, market integration, and relationships between prices: an application to world salmon markets." *American Journal of Agricultural Economics*, 81(3): 568–581.

Axelrod, Robert. 1997. "Advancing the art of simulation in the social sciences." In *Simulating Social Phenomena.* , ed. Rosaria Conte, Rainer Hegselmann and Pietro Terna, 21–40. Springer.

Axtell, Robert. 2000. "Why agents: on the varied motivations for agent computing in social sciences." *Center on Social and Economic Dynamics, The Brookings Institution.*

Ben-Akiva, Moshe, Andre De Palma, and Jacques-Francois Thisse. 1989. "Spatial competition with differentiated products." *Regional Science and Urban Economics*, 19(1): 5–19.

Borrill, Paul L, and Leigh Tesfatsion. 2011. "Agent-based modeling: the right mathematics for the social sciences?" In *The Elgar Companion to Recent Economic Methodology.* , ed. John B Davis and Wade Hands. Edward Elgar Publishing.

Caplin, Andrew, and Barry Nalebuff. 1991. "Aggregation and imperfect competition: On the existence of equilibrium." *Econometrica*, 59(1): 25–59.

Chamberlin, Edward H. 1953. "The product as an economic variable." *The Quarterly Journal of Economics*, 67(1): 1–29.

Chen, Yongmin, and Michael H Riordan. 2008. "Price-increasing competition." *The RAND Journal of Economics*, 39(4): 1042–1058.

d'Aspremont, Claude, J Jaskold Gabszewicz, and J-F Thisse. 1979. "On Hotelling's "Stability in competition"." *Econometrica*, 47(5): 1145–1150.

Devletoglou, Nicos E. 1965. "A dissenting view of duopoly and spatial competition." *Economica*, 32(126): 140–160.

Drezner, Zvi. 1982. "Competitive location strategies for two facilities." *Regional Science and Urban Economics*, 12(4): 485–493.

Eaton, B Curtis, and Richard G Lipsey. 1975. "The principle of minimum differentiation reconsidered: Some new developments in the theory of spatial competition." *The Review of Economic Studies*, 42(1): 27–49.

Fujita, Masahisa, and Jacques-François Thisse. 2013. *Economics of Agglomeration: Cities, Industrial Location, and Globalization.* Cambridge University Press.

Götz, Georg. 2005. "Endogenous sequential entry in a spatial model revisited." *International Journal of Industrial Organization*, 23(3): 249–261.

Gupta, Barnali, Debashis Pal, and Jyotirmoy Sarkar. 1997. "Spatial Cournot competition and agglomeration in a model of location choice." *Regional Science and Urban Economics*, 27(3): 261–282.

Hartwick, John M, and Philip G Hartwick. 1971. "Duopoly in space." *The Canadian Journal of Economics/Revue canadienne d'Economique*, 4(4): 485–505.

Hehenkamp, Burkhard, and Achim Wambach. 2010. "Survival at the center: the stability of minimum differentiation." *Journal of Economic Behavior & Organization*, 76(3): 853–858.

Hotelling, Harold. 1929. "Stability in competition." *The Economic Journal*, 39(153): 41–57.

Irmen, Andreas, and Jacques-Francois Thisse. 1998. "Competition in multi-characteristics spaces: Hotelling was almost right." *Journal of Economic Theory*, 78(1): 76–102.

Jonas, Paul. 1968. "Spatial competition in a two dimensional market." *The Annals of Regional Science*, 2(1): 12–29.

Lerner, Abba Ptachya, and Hans Werner Singer. 1937. "Some notes on duopoly and spatial competition." *The Journal of Political Economy*, 45(2): 145–186.

Lin, C-Y Cynthia, and Gernot Wagner. 2007. "Steady-state growth in a Hotelling model of resource extraction." *Journal of Environmental Economics and Management*, 54(1): 68–83.

Meagher, Kieron J. 2012. "Optimal product variety in a Hotelling model." *Economics Letters*, 117(1): 71–73.

Meagher, Kieron J, and Klaus G Zauner. 2004. "Product differentiation and location decisions under demand uncertainty." *Journal of Economic Theory*, 117(2): 201–216.

Okabe, Atsuyuki, and Masaki Aoyagi. 1991. "Existence of equilibrium configurations of competitive firms on an infinite two-dimensional space." *Journal of Urban Economics*, 29(3): 349–370.

Ottino, B, Forrest Stonedahl, and Uri Wilensky. 2009. "NetLogo Hotelling Model." *Center for Connected Learning and Computer-based Modeling, Northwestern University, Evanston, Il.*

Puu, Tönu. 2002. "Hotelling's Ice cream dealers with elastic demand." *The Annals of Regional Science*, 36(1): 1–17.

Robinson, Austin. 1941. "A problem in the theory of industrial location." *The Economic Journal*, 51(202/203): 270–275.

Roemer, John E. 2009. *Political Competition: Theory and Applications.* Harvard University Press.

Smithies, Arthur. 1941. "Optimum location in spatial competition." *The Journal of Political Economy*, 49(3): 423–439.

Tabuchi, Takatoshi. 1994. "Two-stage two-dimensional spatial competition between two firms." *Regional Science and Urban Economics*, 24(2): 207–227.

Veendorp, ECH, and Anjum Majeed. 1995. "Differentiation in a two-dimensional market." *Regional Science and Urban Economics*, 25(1): 75–83.

Wendell, RE, and RD McKelvey. 1981. "New perspectives in competitive location theory." *European Journal of Operational Research*, 6(2): 174–182.

Wilensky, Uri. 1999. "NetLogo." *Center for Connected Learning and Computer-based Modeling, Northwestern University, Evanston, Il.*